Д/з №1

Найти $v_{гр.}$ и $v_{ф.}$ для е-нов, движ-ся со скор-ю v
1) в релятив. случае
2) в нерелятив. случае

1) $$v_\varphi = \frac{\omega}{k} = \frac{\hbar\omega}{\hbar k} = \frac{\varepsilon}{p} = \frac{\sqrt{c^2p^2+c^4m^2}}{p}$$

$$p = mc\gamma$$

$$v_\varphi = \frac{\sqrt{c^4m^2\gamma^2+c^4m^2}}{mc\gamma} = c\,\frac{\sqrt{\gamma^2+1}}{\gamma},$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$v_{гр} = \frac{d\omega}{dk} = \frac{d\varepsilon}{dp} = \frac{d\left(\sqrt{c^2p^2+c^4m^2}\right)}{dp} = \frac{c^2p}{\varepsilon}$$

$$v_{гр} = \frac{c^2mc\gamma}{mc^2\gamma} = c$$

2) Нерелятив. случай $\quad p_e = m_e V$

$$v_\varphi = \frac{\sqrt{c^2m^2V^2+c^4m^2}}{mV} = \frac{c}{V}\sqrt{V^2+c^2} > c$$

$$v_{гр} = \frac{c^2p}{\varepsilon} = \frac{c^2mV}{\sqrt{c^2m^2V^2+c^4m^2}} = \frac{cV}{\sqrt{V^2+c^2}} < c$$

Д/з №2

Определить время расплывания пакета
а) частицы, массой 1 г, $\Delta x \sim 0{,}1$ см

$$\Delta t = \frac{(\Delta x)^2}{2\pi\hbar\,\frac{d^2E}{dp^2}}$$

Е. нерелятив. частица $E = \frac{p^2}{2m}$

$$\frac{d^2E}{dp^2} = \frac{1}{m}$$

$$\Rightarrow \Delta t = \frac{(\Delta x)^2 m}{2\pi \hbar}$$

$$\Rightarrow \Delta t \sim 10^{25} c$$

б) $\bar{e}$ $m \sim 10^{-27}$ г

$\Delta x \sim 10^{-8}$ см $\Delta t = \frac{(\Delta x)^2 m}{2\pi \hbar}$

$$\Delta t = 10^{-16} c$$

$\boxed{Д/з \ \ №9}$

Получили:

$$\begin{cases} \delta = \operatorname{arcctg}\sqrt{\frac{2mV_1^2}{\hbar^2 k^2} - 1} + n\pi \\ ka + \delta = -\operatorname{arcctg}\sqrt{\frac{2mV_2^2}{\hbar^2 k^2} - 1} + m\pi \end{cases}$$

(Выразить через arcctg и arcsin)

Имеем:

$$\begin{cases} \operatorname{ctg}\delta = \sqrt{\frac{2mV_1}{\hbar^2 k^2} - 1} \\ \operatorname{ctg}(ka + \delta) = -\sqrt{\frac{2mV_2}{k^2\hbar^2} - 1} \end{cases}$$

П. $\operatorname{ctg}\alpha = \sqrt{a^2 - 1}$

$1 + \operatorname{ctg}^2\alpha = \frac{1}{\sin^2\alpha} \Rightarrow 1 + a^2 - 1 = \frac{1}{\sin^2\alpha}$

$\Rightarrow \sin^2\alpha = \frac{1}{a^2}$ $\sin\alpha = \frac{1}{a}$

$$\Rightarrow \sin\delta = \frac{\hbar k}{\sqrt{2mV_1}}$$

$$\sin[-(ka + \delta)] = \frac{\hbar k}{\sqrt{2mV_2}} \Rightarrow \begin{cases} \delta = \arcsin\frac{k\hbar}{\sqrt{2mV_1}} + n\pi \\ ka + \delta = -\arcsin\frac{k\hbar}{\sqrt{2mV_2}} + m\pi \end{cases}$$

$\boxed{\text{Д/з № 4}}$

Определить уровни энергии $\quad n = 1$

$$a\sqrt{2mV} \ll \hbar$$

$$\begin{cases} \arcsin \dfrac{k\hbar}{\sqrt{2mV}} = \dfrac{n\pi}{2} - \dfrac{ka}{2} \\ E = \dfrac{k^2\hbar^2}{2m} \end{cases}$$

$$\begin{cases} \dfrac{k\hbar}{\sqrt{2mV}} = \sin\left(\dfrac{\pi}{2} - \dfrac{ka}{2}\right) \\ k\hbar = \sqrt{2mE} \end{cases}$$

$$\frac{\sqrt{2mE}}{\sqrt{2mV}} = \cos\left(\frac{\sqrt{2mE}}{\hbar}\frac{a}{2}\right) \approx 1 - \frac{\sqrt{2mE}\,a}{2\hbar}$$

$$\sqrt{E}\left(\frac{1}{\sqrt{V}} + \frac{a\sqrt{m}}{\sqrt{2}\,\hbar}\right) = 1$$

$$\Rightarrow \sqrt{E} = \frac{\sqrt{2V}\,\hbar}{\left(\sqrt{2}\,\hbar + a\sqrt{mV}\right)} =$$

$$= \frac{\sqrt{V}}{\left(1 + \dfrac{a\sqrt{mV}}{\sqrt{2}\,\hbar}\right)}$$

$$E = V \frac{1}{1 + \dfrac{a^2mV}{2\hbar^2} + \underbrace{\dfrac{2a\sqrt{mV}}{\sqrt{2}\,\hbar}}_{\to 0}}^{2}$$

$$\approx V\left(1 - \frac{Va^2m}{2\hbar^2}\right) \qquad \text{т.к. } a\sqrt{2mV} \ll \hbar$$

Ответ: $E = V\left(1 - \dfrac{Va^2m}{2\hbar^2}\right)$

Д/з №5.

$$[f(\hat{x}), \hat{p}]\psi = f(\hat{x})\hat{p}\psi - \hat{p}f(\hat{x})\psi =$$

$$= -i\hbar f(\hat{x})\psi' + i\hbar \frac{d}{dx}\left(f(\hat{x})\psi\right) =$$

$$= \cancel{-i\hbar f(\hat{x})\psi'} + i\hbar f'(\hat{x})\psi + \cancel{i\hbar f(\hat{x})\psi'} =$$

$$= i\hbar f'(\hat{x})\psi$$

$$\Rightarrow [f(\hat{x}), \hat{p}] = c f'(\hat{x})$$

Д/з. №6.

$$\hat{\ell}_i = \left([\hat{x}\hat{p}]\right)_i = \varepsilon_{ike} \hat{x}_k \hat{p}_e$$

$$[\hat{\ell}_i \hat{\ell}_k] = [\varepsilon_{ine} \hat{x}_n \hat{p}_e, \varepsilon_{ksr} \hat{x}_s \hat{p}_r] =$$

$$= \varepsilon_{ine}\varepsilon_{ksr} [\hat{x}_n \hat{p}_e, \hat{x}_s \hat{p}_r] = \varepsilon_{ine}\varepsilon_{ksr} [\hat{x}_n [\hat{p}_e, \hat{x}_s \hat{p}_r]$$

$$+ [\hat{x}_n, \hat{x}_s \hat{p}_r]\hat{p}_e] =$$

$$= \varepsilon_{ine}\varepsilon_{ksr}\left(-\hat{x}_n [\hat{x}_s \hat{p}_r, p_e] - [\hat{x}_s \hat{p}_r, \hat{x}_n]\hat{p}_e\right) =$$

$$= \varepsilon_{ine}\varepsilon_{ksr}\left(-\hat{x}_n \hat{x}_s \cancelto{0}{[\hat{p}_r, \hat{p}_e]} - \hat{x}_n [\hat{x}_s \hat{p}_e]\hat{p}_r -\right.$$

$$\left. - \hat{x}_s [\hat{p}_r \hat{x}_n]\hat{p}_e - \cancelto{0}{[\hat{x}_s, \hat{x}_n]}\hat{p}_r \hat{p}_e\right) =$$

$$= i\hbar \varepsilon_{ine}\varepsilon_{ksr}\left(-\hat{x}_n \hat{x}_s \delta_{se} + \hat{x}_s \hat{p}_e \delta_{nr}\right) =$$

$$= -i\hbar \varepsilon_{ins}\varepsilon_{ksr}\hat{x}_n \hat{p}_r + i\hbar \varepsilon_{ine}\varepsilon_{ksn}\hat{x}_s \hat{p}_e =$$

$$= i\hbar\left(\varepsilon_{sin}\varepsilon_{skr}\hat{x}_n \hat{p}_r - i\hbar\, \varepsilon_{nie}\varepsilon_{nks}\hat{x}_s \hat{p}_e\right] =$$

$$= i\hbar\left(\delta_{ik}\delta_{nr}\hat{x}_n \hat{p}_r - \delta_{ir}\delta_{nk}\hat{x}_n \hat{p}_r - \delta_{ik}\delta_{es}\hat{x}_s \hat{p}_e +\right.$$

$$\left. + \delta_{is}\delta_{ek}\hat{x}_s \hat{p}_e\right) = i\hbar\left(\cancel{\delta_{ik}\hat{x}_n \hat{p}_n} - \delta_{ir}\delta_{nk}\hat{x}_n \hat{p}_r -\right.$$

$$\left. - \cancel{\delta_{ik}\hat{x}_s \hat{p}_s} + \delta_{is}\delta_{ek}\hat{x}_s \hat{p}_e\right] = -\hat{x}_k \hat{p}_i + \hat{x}_i \hat{p}_k =$$

$$= \varepsilon_{ikr}\ell_r i\hbar$$

$\boxed{\text{Д/з №7}}$

$[\hat{P}_n, \hat{P}_m] = ?$ $\qquad \hat{P} = \hat{p} - \frac{e}{c}\hat{A}$

$\hat{A}(r) = \{\hat{A}_x(r), \hat{A}_y(r), \hat{A}_z(r)\}$

$$[\hat{p}_n - \frac{e}{c}\hat{A}_x, \hat{p}_m - \frac{e}{c}\hat{A}_y] =$$

$$= [\cancel{\hat{p}_n, \hat{p}_m}]^{\to 0} - \frac{e}{c}\left([\hat{p}_n \hat{A}_y] + [\hat{A}_x \hat{p}_m]\right) +$$

$$+ \frac{e^2}{c^2}[\hat{A}_x \hat{A}_y] =$$

$$= -\frac{e}{c}[\hat{p}_n \hat{A}_y \psi - \hat{A}_y \hat{p}_n \psi + \hat{A}_x \hat{p}_m \psi - \hat{p}_m \hat{A}_x \psi] +$$

$$+ \frac{e^2}{c^2}\underbrace{(\hat{A}_x \hat{A}_y \psi - \hat{A}_y \hat{A}_x \psi)}_{0} =$$

$$= -\frac{e}{c}\left[\frac{d}{dx}(\hat{A}_y \psi) - \hat{A}_y \frac{d}{dx}\psi + \hat{A}_x \frac{d}{dy}\psi - \right.$$

$$\left. - \frac{d}{dy}(\hat{A}_x \psi)\right] =$$

$$= -\frac{e}{c}\left[\hat{A}_y \frac{d}{dx}\psi - \hat{A}_y \frac{d\psi}{dx} + \hat{A}_x \frac{d\psi}{dy} - \hat{A}_x \frac{d\psi}{dy}\right] =$$

$$= 0$$

$\boxed{\text{Д/з №12}}$

Вычислить $\langle n | \hat{x} \hat{p} | m \rangle$ исп-я „a^+ и a"

$$\langle n|\hat{x}\hat{p}|m\rangle = \left\| \begin{array}{l} \hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a}+\hat{a}^+) \\ \hat{p} = \frac{\sqrt{2m\omega\hbar}}{2i}(\hat{a}^+-\hat{a}) \end{array} \right\| =$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\frac{\sqrt{2m\omega\hbar}}{2i} \langle n|(\hat{a}+\hat{a}^+)(\hat{a}^+-\hat{a})|m\rangle =$$

$$\frac{\hbar}{2i}\langle n|\hat{a}\hat{a}^+ - \hat{a}\hat{a} + \hat{a}^+\hat{a}^+ - \hat{a}^+\hat{a}|m\rangle =$$

$$\frac{\hbar}{2i}\Big(\langle n|\hat{a}\hat{a}^+|m\rangle - \langle n|\hat{a}\hat{a}|m\rangle + \langle n|\hat{a}^+\hat{a}^+|m\rangle - \langle n|\hat{a}^+\hat{a}|m\rangle\Big) =$$

$$\frac{\hbar}{2i}\Big(\langle n|\sqrt{m+1}\,\hat{a}|m+1\rangle - \langle n|\hat{a}\sqrt{m}|m-1\rangle +$$

$$+ \langle n|\hat{a}^+\sqrt{m+1}|m+1\rangle - \langle n|\hat{a}^+\sqrt{m}|m-1\rangle\Big) =$$

$$= \frac{\hbar}{2i}\Big(\sqrt{m+1}\cdot\sqrt{m+1}\,\langle n|m\rangle - \sqrt{m}\sqrt{m-1}\,\langle n|m-2\rangle +$$

$$\sqrt{m+1}\sqrt{m+2}\,\langle n|m+2\rangle - \sqrt{m}\sqrt{m}\,\langle n|m\rangle\Big) =$$

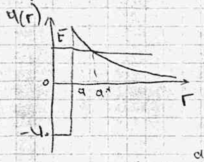
$$= \frac{\hbar}{2i}\Big((m+1)\delta_{nm} - \sqrt{m}\sqrt{m-1}\,\delta_{n,m-2} +$$

$$+ \sqrt{m+1}\sqrt{m+2}\,\delta_{n,m+2} - m\,\delta_{nm}\Big) =$$

$$= \frac{\hbar}{2i}\Big(\delta_{nm} - \sqrt{m}\sqrt{m-1}\,\delta_{n,m-2} + \sqrt{m+1}\sqrt{m+2}\,\delta_{n,m+2}\Big)$$

Д/з №7

Туннельный эффект



$$U(r) = \begin{cases} -U_0, & r \le a \\ \dfrac{2(Z-2)e^2}{r}, & r \ge a \end{cases}$$

определить к-т пропускания D

$$D \sim \exp\left\{-\frac{2}{\hbar}\int_a^{a^*} p\,dr\right\} = \exp\left\{-\frac{2}{\hbar}\int_a^{a^*}\sqrt{2m(E+U(r))}\,dr\right\}$$

$$= \exp\left\{-\frac{2}{\hbar}\int_a^{a^*}\sqrt{2m\left(\frac{2(Z-2)e^2}{r}-E\right)}\,dr\right\} =$$

$$\| A = 2(Z-2)e^2 = \text{const} \|$$

$$= \exp\left\{-\frac{2\sqrt{2m}}{\hbar}\int_a^{a^*}\sqrt{\frac{A}{r}-E}\,dr\right\} =$$

$$= \exp\left\{-\frac{2\sqrt{2m}}{\hbar}\int_a^{a^*}\frac{\sqrt{A-Er}}{\sqrt{r}}\,dr\right\} =$$

$$= \exp\left\{-2\frac{2\sqrt{2m}}{\hbar}\int_a^{a^*}\sqrt{A-Er}\,d(\sqrt{r}) = \|\sqrt{r}=t\| =\right.$$

$$= \exp\left\{-\frac{4\sqrt{2m}}{\hbar}\int_{\sqrt{a}}^{\sqrt{a^*}}\sqrt{A-Et^2}\,dt =\right.$$

$$= \exp\left\{-\frac{4\sqrt{2mE}}{\hbar}\int_{\sqrt{a}}^{\sqrt{a^*}}\sqrt{\frac{A}{E}-t^2}\,dt\right\} =$$

$$= \exp\left\{ -\frac{4\sqrt{2mE}}{\hbar}\left[\frac{t}{2}\sqrt{\left(\frac{A}{E}\right)^2 - t^2}\,\Bigg|_{\sqrt{a}}^{\sqrt{a^*}} - \frac{\left(\frac{A}{E}\right)^2}{2} \otimes \right.\right.$$

$$\otimes \left.\left. \arcsin \frac{t\sqrt{E}}{\sqrt{A}}\Bigg|_{\sqrt{a}}^{\sqrt{a^*}} \right] + C \right\} =$$

$$= \exp\left\{ -\frac{4\sqrt{2m}}{\hbar}\left[\frac{\sqrt{a^*}}{2}\sqrt{A - a^*E} - \frac{\sqrt{a}}{2}\sqrt{A - aE} - \right.\right.$$

$$\left.\left. - \frac{A}{2E}\left(\arcsin\sqrt{\frac{Ea^*}{A}} - \arcsin\sqrt{\frac{Ea}{A}}\right)\right] + C \right\} =$$

$$= \exp\left\{ -\frac{2}{\hbar}\left(a^* p(a^*) - a\cdot p(a)\right) + \right.$$

$$\left. + \frac{2\sqrt{2m}A}{E\hbar}\left(\arcsin\sqrt{\frac{Ea^*}{A}} - \arcsin\sqrt{\frac{Ea}{A}}\right) + C \right\} \circleddash$$

В т. a^* $\quad U(a^*) = E \Rightarrow p(a^*) = 0$

$$\frac{A}{a^*} = E \Rightarrow \arcsin\sqrt{\frac{Ea^*}{A}} = \arcsin 1 = \frac{\pi}{2}$$

$$= \exp\left\{ \frac{2}{\hbar} a\,p(a) + \frac{2\sqrt{2m}A}{E\hbar}\left(\frac{\pi}{2} - \arcsin\sqrt{\frac{Ea}{A}}\right) + C \right\}$$

№ 11

Выразить $\hat{p}^2$ и $\hat{V}$ через $\hat{a}$ и $\hat{a}^+$ для осциллятора

$$\hat{p} = \frac{\sqrt{2m\omega\hbar}}{2i}\left(\hat{a}^+ - \hat{a}\right), \quad \hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a} + \hat{a}^+\right)$$

$$\hat{p}^2 = -\frac{2m\omega\hbar}{4}\left((\hat{a}^+)^2 + (\hat{a})^2 - \hat{a}\hat{a}^+ - \hat{a}^+\hat{a}\right)$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 x^2}{2} = \hbar\omega\left(\hat{a}^+\hat{a} + \frac{1}{2}\right)$$