1. $\hat\rho = \sum_n \omega_n |\Psi_n\rangle\langle\Psi_n|$

а) $\hat\rho^+ = \hat\rho$ б) $\langle m|\hat\rho|m\rangle = \sum_n \omega_n |C_{nm}|^2$

в) $Tr\,\hat\rho = 1$, г) $Tr\,\hat\rho^2 \underset{\text{чист}}{=} 1 \underset{\text{смеш}}{<} 1$ д) чист. сост: $\hat\rho^2 = \hat\rho$

$i\hbar \frac{\partial}{\partial t}\hat\rho(t) = [\hat H, \hat\rho]$

2. $\langle A\rangle = \sum_n \omega_n \langle A_n\rangle$

$\langle A_n\rangle = \langle\Psi_n|A|\Psi_n\rangle$

3. Если $[\hat A, \hat B] = i\hat C$,

$\langle(\Delta\hat A)^2\rangle\langle(\Delta\hat B)^2\rangle \geq \frac{1}{4}\langle\hat C\rangle^2$,

$\Delta\hat A = \hat A - \langle\hat A\rangle\hat I$, $\langle\hat A\rangle = \langle\Psi|\hat A|\Psi\rangle$

$\Delta p^2 \Delta x^2 \geq \frac{\hbar^2}{4}$, $[\hat p_i, \hat x_i] = -i\hbar$

5. $|\Psi_r\rangle = \hat S^{-1}(t,t_0)|\Psi(t)\rangle$, $\hat S: |\Psi(t)\rangle = \hat S(t,t_0)|\Psi(t_0)\rangle$

$|\Psi_{ez}(t)\rangle = \hat S_0^{-1}|\Psi\rangle$, $\hat S_0 = e^{-\frac{i}{\hbar}H_0(t-t_0)}$ а) $\hat S(t_1,t_2)\cdot\hat S(t_2,t_3) = \hat S(t_1,t_3)$

б) $\hat S$-унитарен, т.е. $\hat S^+ = S^{-1}$

$\hat A_r = \hat S^{-1}\hat A\hat S$

$\hat A_{ez} = \hat S_0^{-1}\hat A\hat S_0$ Шр: $i\hbar\frac{\partial}{\partial t}|\Psi_{\text{Ш}}\rangle = \hat H|\Psi_{\text{Ш}}\rangle$; $\frac{\partial\hat x}{\partial t} = 0$, $\frac{\partial\hat p}{\partial t} = 0$

Гейз: $\frac{\partial}{\partial t}|\Psi_r\rangle = 0$; $\frac{\partial\hat A_r}{\partial t} = \hat S^{-1}\frac{\partial A}{\partial t}\hat S + \frac{i}{\hbar}[\hat H, \hat A_r]$

Взаим: $i\hbar\frac{\partial}{\partial t}|\Psi_{ez}\rangle = \underset{\hat H - \hat H_0}{\hat V_{ez}}|\Psi_{ez}\rangle$, $\frac{\partial \hat A_{ez}}{\partial t} = \hat S_0^{-1}\frac{\partial A}{\partial t}\hat S + \frac{i}{\hbar}[H_0, \hat A_{ez}]$

6. $i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi + U(\vec r, t)\Psi$, где: $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$;

$\Delta = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}$; $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$

7. $\rho = \psi^*\psi$, $\vec{j} = \frac{i\hbar}{2m}\left[\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right]$

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\vec{j} = 0$$

8. $\left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - \frac{e}{c}\vec{A}\right)^2 + e\varphi\right]\psi = i\hbar\frac{\partial}{\partial t}\psi$

9. Поле $\vec{H} = H_0\vec{e}_z$

$$\left[-\frac{\hbar^2}{2m}\Delta - \frac{e}{2mc}\left(\vec{H}\vec{L}\right) + \frac{eH_0^2}{8mc^2}(x^2+y^2)\right]\psi = E\psi$$

10. $\psi(t,x) \approx e^{i/\hbar(S_0 + \hbar S_1)}$, $S_0 = -Et + \int p\,dx$

точки поворота - не примен. $S_1 = \frac{i}{2}\ln p(x)$

11. $T = e^{-2/\hbar\int_{x_1}^{x_2}|p|dx}$; $x_1, x_2 : U(x) = E$

12. $\frac{1}{\hbar}\int_{x_1}^{x_2} p\,dx = \pi\left(n + \frac{1}{2}\right)$; $\frac{1}{2\hbar}\oint p\,dx = \pi\left(n + \frac{1}{2}\right)$

13. $\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega^2x^2}{2}\right)\psi = E\psi$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\hat{a}^+ = \frac{1}{\sqrt{2}}\left(\xi - \frac{d}{d\xi}\right), \quad \hat{a} = \frac{1}{\sqrt{2}}\left(\xi + \frac{d}{d\xi}\right)$$

$[\hat{a}, \hat{a}^+] = 1$; $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$, $\hat{a}^+|n\rangle = \sqrt{n+1}|n+1\rangle$

$$H = \hbar\omega\left(\hat{a}^+\hat{a} + 1/2\right)$$

14. $\hat{L}^2 = -\hbar^2\Delta_{\theta\varphi}$ $\hat{L}_z = \frac{i}{\hbar}\frac{\partial}{\partial\varphi}$

15. $\left(-\frac{\hbar^2}{2m}\Delta + U(|\vec{r}|)\right)\psi = E\psi$

$\Psi_{lm} = e^{-\frac{E}{\hbar}ti}\, Y_{lm}(\theta,\varphi) R_l(r)$, $Y_{lm}(\theta,\varphi)$ – сфер. функц.

$$-\frac{\hbar^2}{2mr^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{\hbar^2 l(l+1)}{2mr^2}R+U(r)R=ER$$